Strain and stress in one dimension

1. Normal strain
2. Shear strain
3. Measurement of strain
4. Definition of normal and shear stress
5. Method of sections
6. Hookes Law
7. Axially loaded bars

Stiffness vs Strength

Stiffness governs the amount a structure is deformed as a result of a load

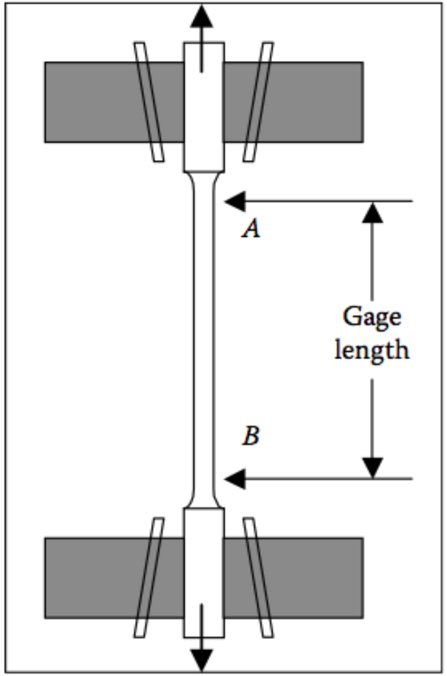
Strength governs how much load a structure can hold without failing

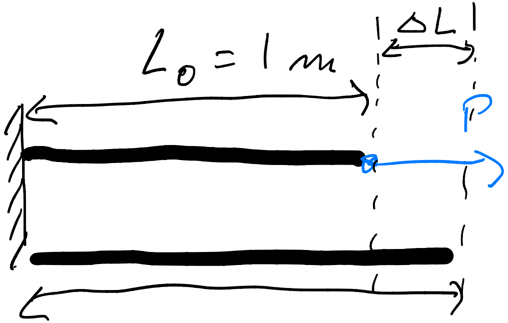
Strain: the deformation of a structure due to a load normalized to it's initial size and shape

Gauge length: initial, undeformed length on the structure of interest

Tensile forces: Forces that tend to stretch or elongate the structure

Compressive forces: Forces that tend to compress or shorten the structure





$L = 1.1\,m$

STRAIN $\varepsilon = \dfrac{\Delta L}{L_0} = \dfrac{L - L_0}{L_0}$

$= \dfrac{1.1\,\cancel{m} - 1\,\cancel{m}}{1\,\cancel{m}}$

$\boxed{\varepsilon = 0{,}1} = 10\,\%$

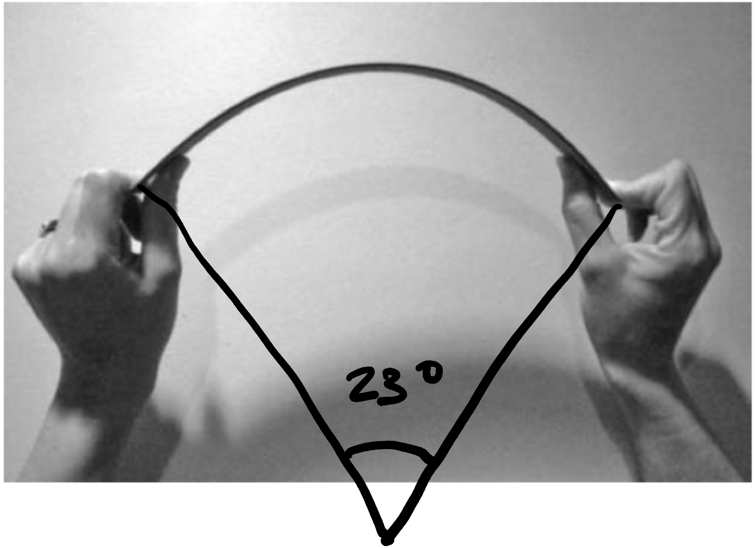
UNIT OF $\varepsilon = 1$

Definition of macroscopic strain

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{\Delta L}{L_0}$$

$\varepsilon > 0$ for tensile forces

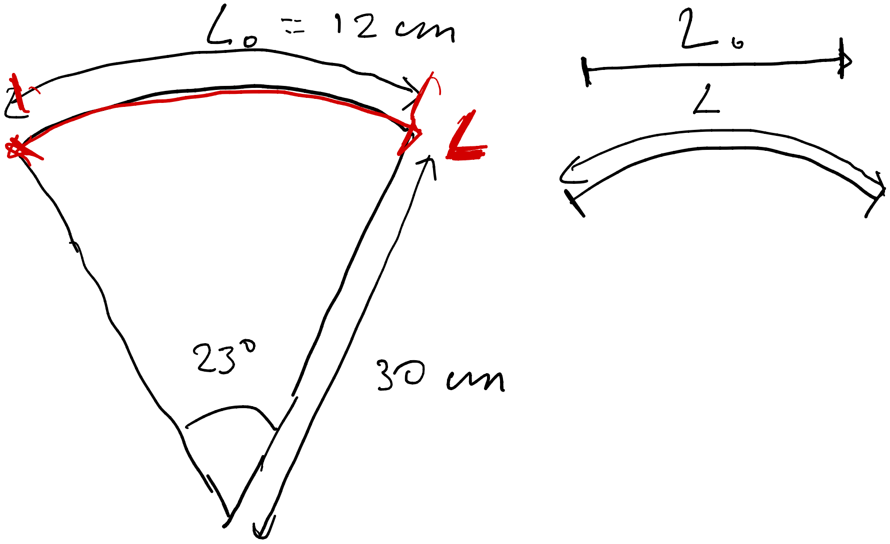
$\varepsilon < 0$ for compressive forces



Example: Strain in a bent ruler

A thin 12cm long ruler is deformed into a circular arc with radius 30cm. In the deformed state, the ruler makes an arc length encompassing 23degr. What is the strain in the ruler?

Assumptions:
ruler length: 12cm
deformation into a circular arc r=30cm
enclosed angle = 23 degree
ruler is thin!
Asked:
find maximum normal strain in ruler



GIVEN: $L_0 = 12$ cm
$\phi = 23°$
$r = 30$ cm

ASKED: STRAIN IN BENT RULER: ε

ASSUMPTIONS: RULER MAKES CIRCULAR ARC.

GOVERNING EQUATIONS

$$\varepsilon = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}$$

ANSWER:

L = ARCLENGTH = $r \cdot \phi$

$$= 30\,\text{cm} \cdot 23° \cdot \frac{2\pi\,\text{rad}}{360°}$$

$$= 12.04277\ldots\ \text{cm}$$

$$\varepsilon = \frac{12.04277 - 12}{12}$$

$$= 3.564 \cdot 10^{-3} = \boxed{3.6 \cdot 10^{-3}}$$

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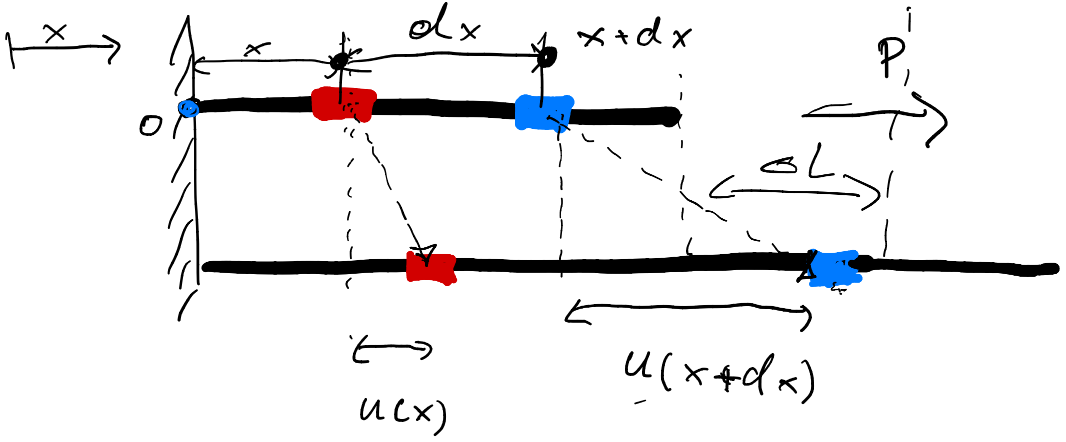
Definition of microscopic strain



$u(x)$: THE DISTANCE THAT A PLANAR SECTION HAS MOVED IN THE DIRECTION OF ELONGATION DUE TO THE LOAD

TAYLOR: $f(z) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(z-a)$

1^{ST} ORDER: $f(z) = f(a) + f'(a)(z-a)$

WHAT IS ε OF SECTION dx:

$$\varepsilon = \frac{L - L_0}{L_0} =$$

$L_0 = dx$

$L = dx + u(x+dx) - u(x)$

$f \Leftrightarrow u$

$z \Leftrightarrow x+dx$

$a \Leftrightarrow x$

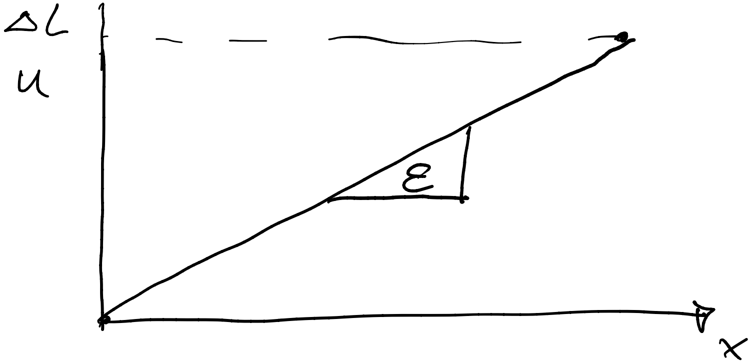
$$\Rightarrow u(x+dx) = u(x) + u'(x)(\cancel{x} + dx - \cancel{x})$$
$$= u(x) + u'(x) \cdot dx$$

Definition of microscopic strain

$$L = dx + \cancel{u(x)} + u'(x)\cdot dx - \cancel{u(x)}$$

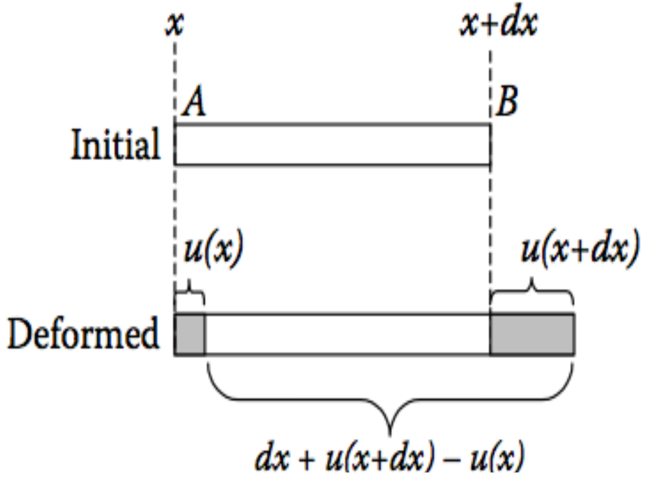
$$\boxed{\varepsilon = \frac{u'(x)\,\cancel{dx} + \cancel{dx} - \cancel{dx}}{\cancel{dx}} = u'(x) = \frac{du}{dx}}$$

LOCAL STRAIN IS THE FIRST DERIVATIVE OF THE DISPLACEMENT!



Definition of microscopic strain

u(x):= the distance a planar section has moved in the direction of the elongation due to the load



$$\varepsilon = \frac{\text{change in length}}{\text{original length}} = \frac{\left[dx + u(x+dx) - u(x)\right] - dx}{dx}$$

Taylor series expansion of u(x+dx)

$$\varepsilon \cong \frac{[u(x) + u'(x)dx - u(x)]}{dx} = u'(x) = \frac{du}{dx}$$

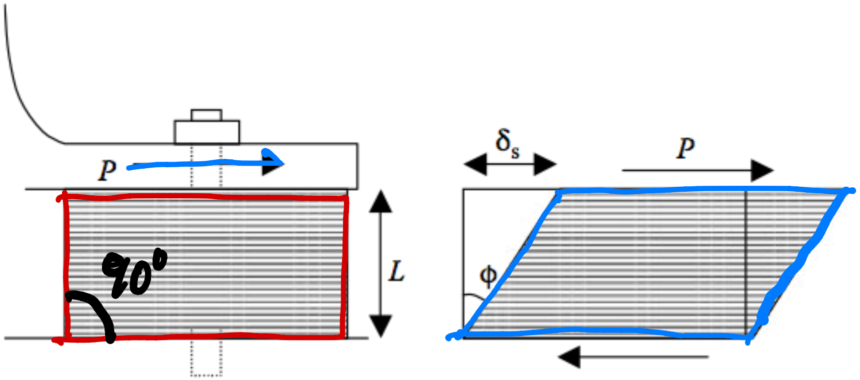
Shear strain

- Shear strain: angular distortion as a result of an applied load

definitions:

Engineering shear strain: the angular change (phi) in an initially right angle due to an applied force

For small deformations we approximate phi by tan(phi)



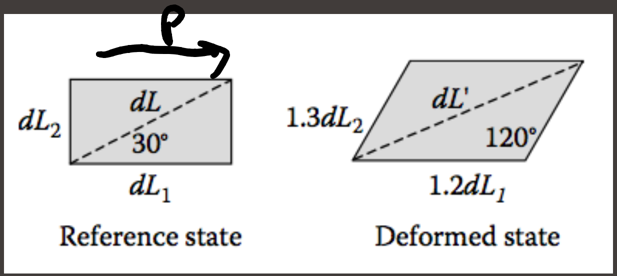
(a) Motor mount (b) Motor mount distorted in shear

$$\phi \cong \tan\phi = \frac{\delta_s}{L}$$

$\phi << 1$

$$\boxed{\gamma \cong \frac{\delta_s}{L}}$$

Example: Shear strain

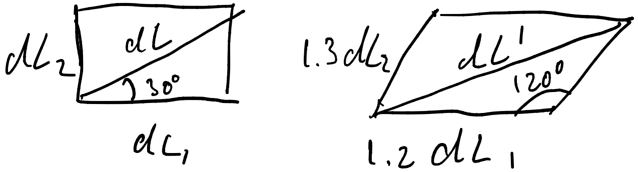


An infinitesimal rectangle at a point in a reference state of a material becomes a parallelogram shown in ta deformed state. Determine:

a) The extensional strain in the dL_1 direction

B) The extensional strain in the dL_2 direction

C) The shear strain coresponding to the dL_1 and dL_2 directions.



given: GEOMETRY BEFORE & AFTER

ASKED: • ε_{L_1} , ε_{L_2}

• γ_{L_1, L_2}

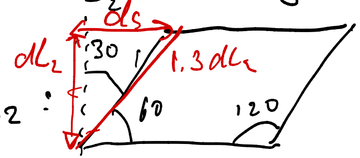
EQN: $\varepsilon = \frac{\Delta L}{L}$ $\qquad \gamma = \tan\phi = \frac{\delta_s}{L}$

ANSWER:

$$\varepsilon_{L_1} = \frac{dL_1' - dL_1}{dL_1} = \frac{1.2\,dL_1 - dL_1}{dL_1} = 0.2$$

$\varepsilon_{L_2} = 0.3$

$\gamma_{1,2}$:



$\gamma_{12} = 30^\circ = \frac{\pi}{6}$ rad $= 0.52$ rad

$$\gamma_{12} = \frac{\delta_s}{dL_2} = \frac{1.3\,dL_2 \cdot \sin 30}{dL_2} = 0.65 \text{ rad}$$

$$GF = \frac{\frac{\Delta R}{R}}{\varepsilon} = \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L_0}}$$

Measurement of strain: strain gauges

Strain gauges change their resistance as a function of strain

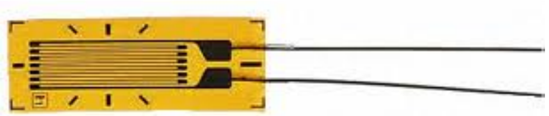
strain gauges can be made of different materials:

- metal wire
- metal thin film
- semiconductors (especially doped silicon)
- nanogranular materials
- …

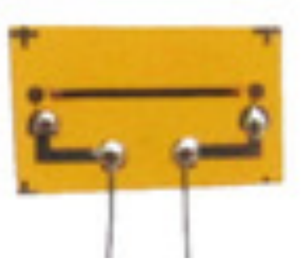
Gauge factor (GF): relative change in resistance due to strain

Different types of strain gauges

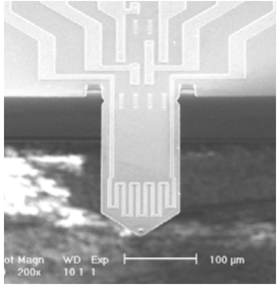
Foil strain gauge, GF=ca 2

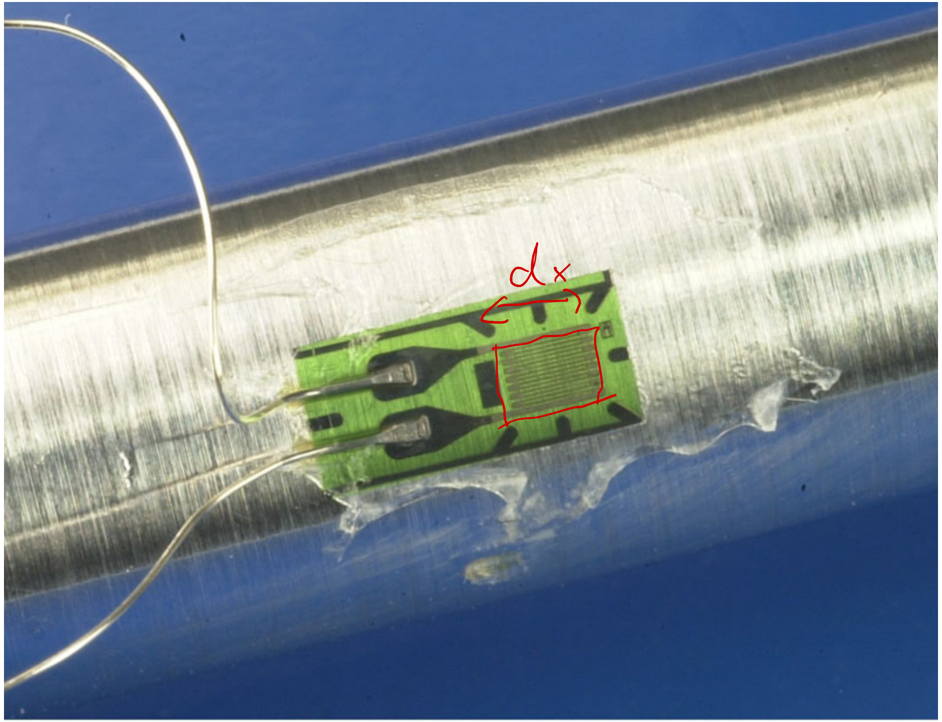


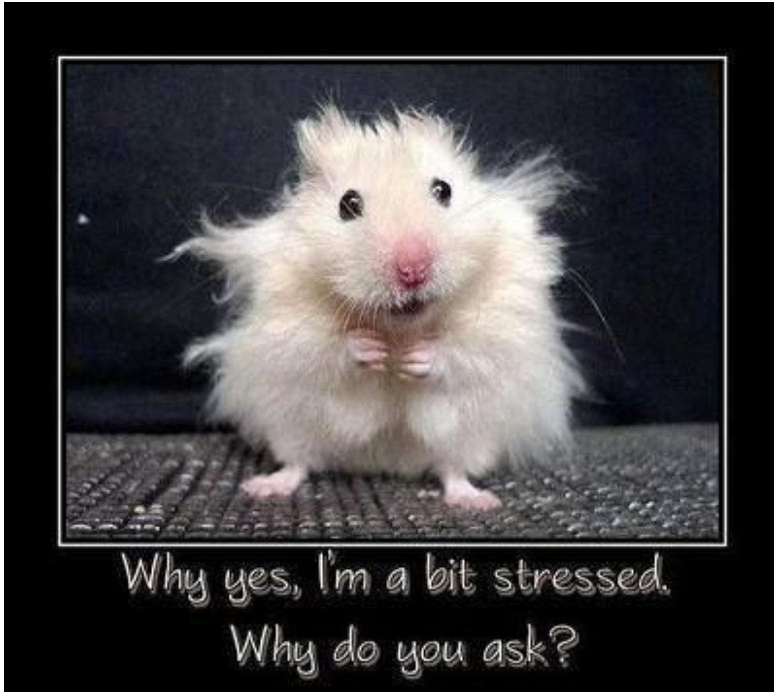
semiconductor strain gauge GF=ca 100



MEMS cantilever with embedded doped Si strain gauges GF=ca 30



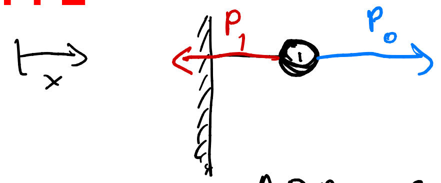




Stress

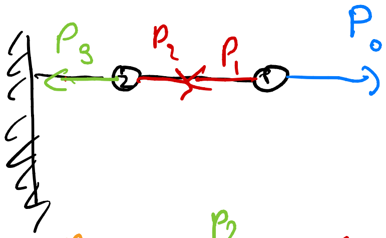
The internal response of structures to external loads

ATOM 1 TETHERED TO WALL

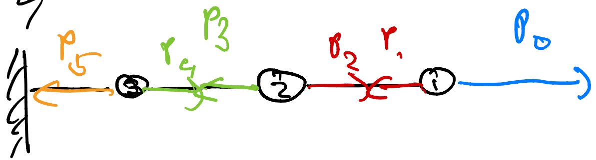


$$\sum F = P_0 - P_1 = 0 \iff P_0 = P_1$$

ADD SECOND ATOM.



$$\sum F = -P_3 + P_2 - \cancel{P_1} + \cancel{P_0} = 0 \Rightarrow P_2 = P_3$$

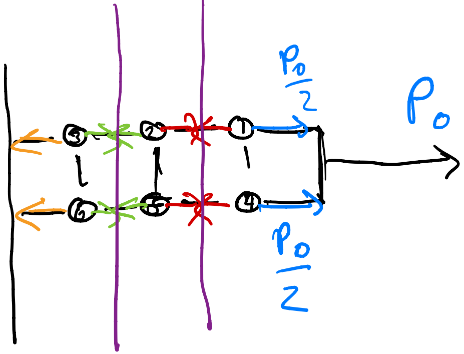


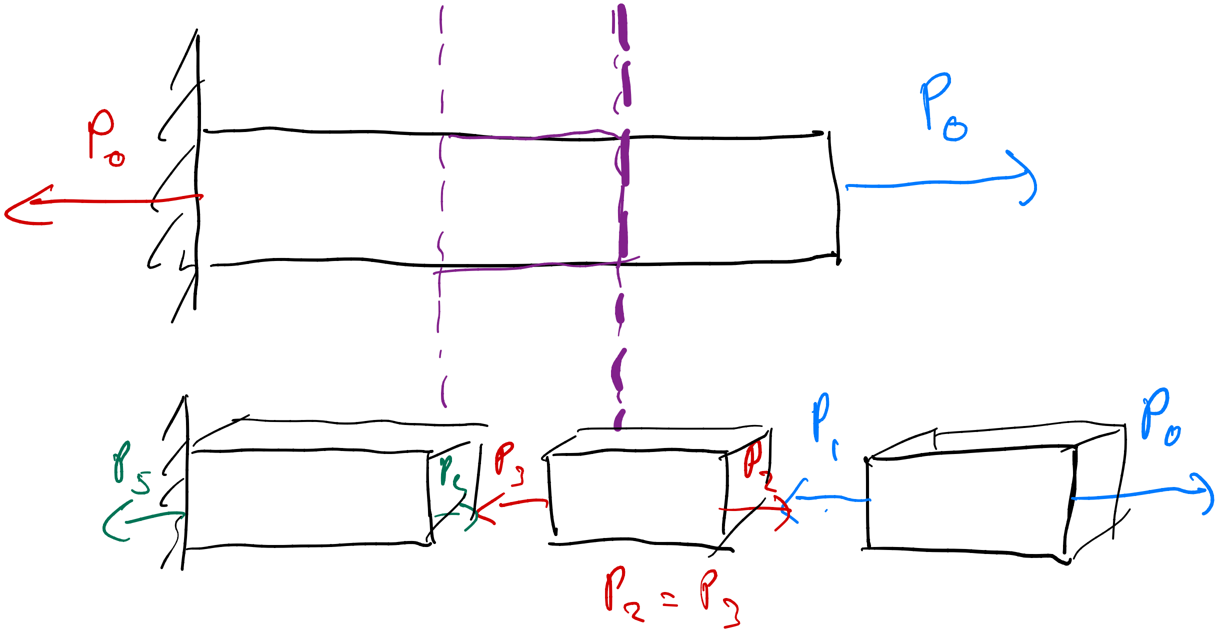
$$\sum F = -P_5 + P_4 - P_3 + P_2 - P_1 + P_0 = 0$$

$$\underbrace{(P_4 - P_5)}_{0} + \underbrace{(P_2 - P_3)}_{0} + \underbrace{(P_0 - P_1)}_{0} = 0$$

IF WHOLE STRUCTURE IS IN EQUILIBRIUM

$\Rightarrow$ EACH ATOM BY ITSELF HAS TO BE IN EQUILIBRIUM



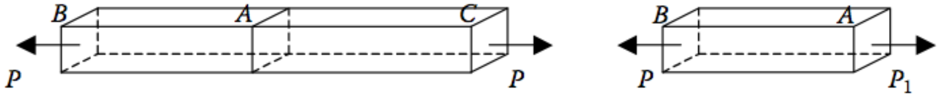


$P_1 = P_0$

Stress = force per unit area

- Normal stress at cross section A:

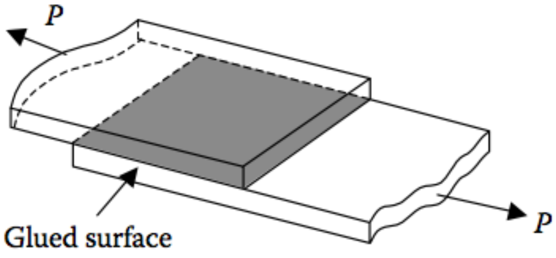
$$\sigma \equiv \frac{P}{A_{\perp}}$$



(a) Bar BC (b) Free body BA

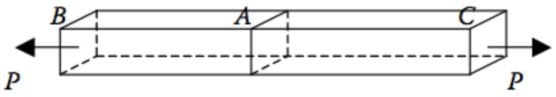
- Shear stress at cross section A:

$$\tau \equiv \frac{P}{A_{\parallel}}$$

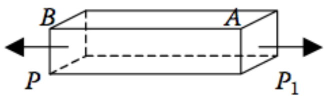


Method of sections

- Free body diagram (FBD): a schematic drawing with all the forces that act on the structure, including reaction forces through supports
- Method of sections:
  - perform a “virtual cut” through the body
  - each of the remaining pieces needs to be by itself in equilibrium. The internal forces that counter the external forces at that section are also present inside the whole body.



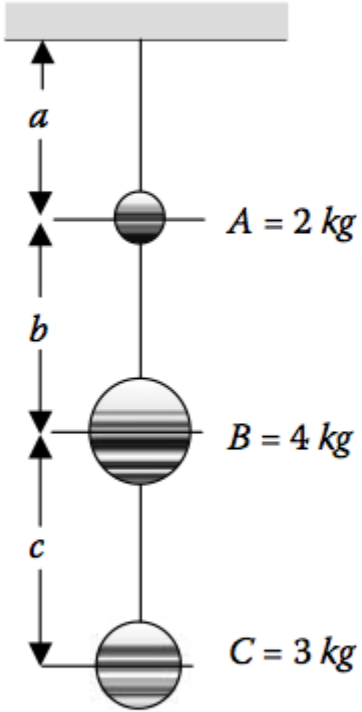
(a) Bar BC

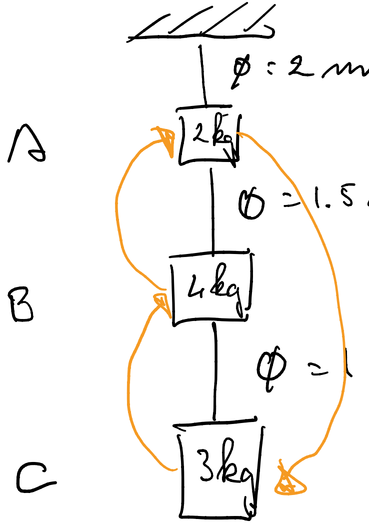


(b) Free body BA

Example: Stress in wires

- Three metal balls are suspended by three wires of equal length arranged in sequence as shown in the figure. The masses of the balls, starting at the top, are 2 kg, 4 kg, and 3 kg, respectively. In the same order, beginning at the top, the wires have diameters 2 mm, 1.5 mm, and 1 mm, respectively.
- (a) Determine the highest stressed wire, and
- (b) by changing the location of the balls, optimize the mass locations to achieve a system with minimum stresses.





$\phi = 2\,mm \Rightarrow r = 1\,mm$

$\phi = 1.5\,mm \Rightarrow r = 0{,}75\,mm$

$\phi = 1\,mm \Rightarrow r = 0{,}5\,mm$

GIVEN:
- MASSES
- DIAMETERS
- ORDER OF ARRANGEMENT

ASKED: STRESS IN WIRES.

ASSUMPTIONS: IGNORE MASS OF WIRES

GOV. EQN: $\sigma = \frac{P}{A}$

ANSWER:

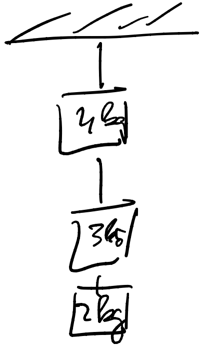
$$\sigma_A = \frac{9\,kg \cdot g}{\pi r^2} = \frac{9 \cdot 9.8}{\pi (1 \cdot 10^{-3})^2} = 28 \cdot 10^6 \frac{N}{m^2}$$

$$= 28\,MPa$$

$$\sigma_B = \frac{7 \cdot g}{\pi r^2} = 38{,}8\,MPa$$

$$\sigma_C = 37.4\,MPa$$

MINIMUM STRESS



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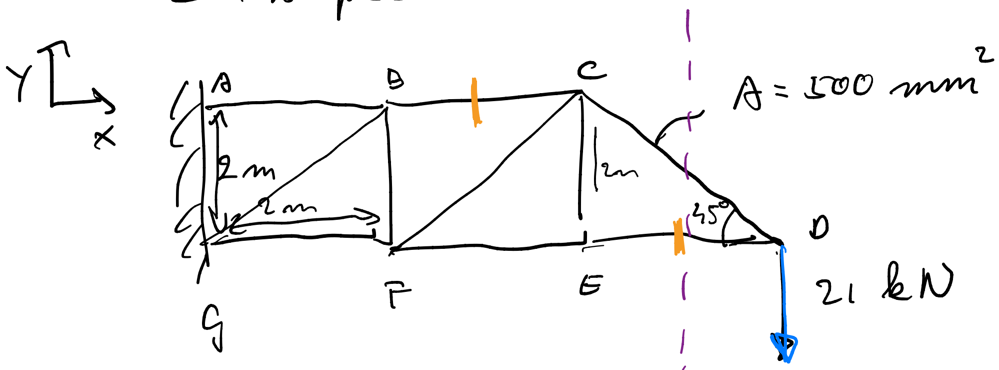
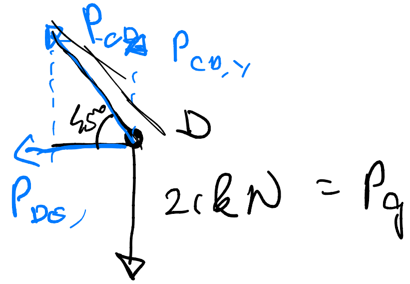
Example : TRUSSES



Given : • geometry
• Load : 21 kN
• A = 500 mm²

Asked : σ_{BC} , σ_{DG}

Gov. princ. : $\sigma = \frac{P}{A}$ & METHOD OF SECTION

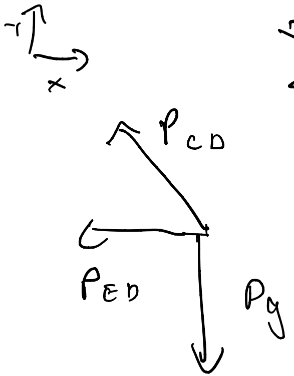


$\sum F = 0$

$\sum F_Y = -P_g + P_{CD,Y}$

$= -P_g + \frac{1}{2}\sqrt{2}\, P_{CD} = 0$

$P_{CD} = \sqrt{2} \cdot P_g$



$$\sum F_x = -P_{ED} - P_{CD,x} = -P_{ED} - \frac{1}{2}\sqrt{2}\cdot P_{DC} = 0$$

$$-P_{ED} - \frac{1}{2}\sqrt{2}\sqrt{2}\,P_g = 0$$

$$\Rightarrow P_{ED} = -P_g = -21\,kN$$

$$\sigma_{ED} = \frac{P_{ED}}{A} = -\frac{21\,kN}{500\cdot 10^{-6}\,m^2} = -42\,MPa$$

$\sigma < 0 \Rightarrow$ COMPRESSION

$\sigma > 0 \Rightarrow$ TENSION

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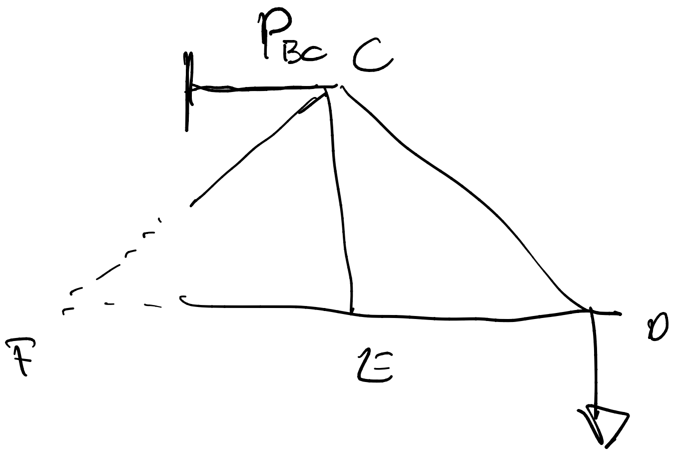

$\sum F = 0$

$\sum M = 0$